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ABELARD'S ARGUMENT FOR FORMALITY

1 Introduction

Buridan defines a formal consequence as one in which no categorematic terms occur essentially—one that remains valid no matter what the matter, provided we keep the form the same¹. A material consequence, by contrast, is one that fails to hold «in all terms, keeping the form alike». So, for example,

(1) A man runs. Therefore an animal runs.

is a material, but not a formal consequence, because the consequence can be destroyed by substituting «horse» for «man» and «wood» for «animal». Similar definitions of formal and material consequence can be found in Pseudo-Scotus and Albert of Saxony².

Two things are striking about this medieval definition. The first is that it is very close to the modern conception of formal consequence one finds in Bolzano and Tarski³. The second is that although Buridan and the other fourteenth-century logicians state these distinctions with a great deal of precision, they say almost nothing about the point of the distinctions. Why are these distinctions drawn in the way they are? What philosophical purpose do they serve?

Although the resemblance of the medieval distinction to the modern one has been widely noted, the lack of explicit motivation has not. I suspect that is because we find the distinction so familiar and natural that we do not pause to think about what motivates it. But the question should be a live one for us, as well. We can sort good inferences into those that can be turned into bad ones by uniform substitution of nonlogical vocabulary, and those that cannot. But what is so special about the inferences in the latter class, the ones we call «formally valid»? Consider this pair:

¹J. BURIDAN, *Tractatus de Consequentibus*, ed. H. HUBIEN (Louvain: Publications Universitaires, 1976), I.4

²I. M. BOCHENSKI, *A History of Formal Logic*, trans. I. THOMAS (Notre Dame: Notre Dame University Press, 1961)

³B. BOLZANO, *Wissenschaftslehre*, 2nd ed. (Leipzig: Felix Meiner, 1929); A. TARSKI, «The Concept of Truth in Formalized Languages», in *Logic, Semantics, Metamathematics*, ed. J. CORCORAN (Indianapolis: Hackett, 1983), 152–278

(2) Snow is white, everything white is colored; therefore, snow is colored.

(3) Snow is white. Therefore, snow is colored.

Is my knowledge that (2) is a good inference any more certain or apriori than my knowledge that (3) is a good inference? Presumably not. Why, then, do we care about the distinction between formally and materially valid inferences? Is it just that the formally valid inferences are more amenable to systematic treatment? But then, thinking of them as valid in a special way—formally valid—would be akin to thinking of stars that can be studied using terrestrial telescopes as a special kind of stars—telescopically accessible stars⁴.

Those who want to avoid this deflationary conception might be tempted to something like the following line of thought. An inference like (3) owes its validity to a fact about the world—the fact that whatever is white is colored. Not a very exciting fact, perhaps, and one that can be known apriori—but a fact nonetheless. In contrast, the validity of (2) does not depend on any fact about white things, snow, or anything else. (2) is valid entirely in virtue of its form or construction; its validity does not depend on any fact about the world, however general. Generalizing, we might say that formally valid inferences are inferences whose validity is not grounded in any fact about the world. Of course, the claim that there *are* such inferences is a substantive one. Let us call it the

Formality Thesis: There are inferences whose validity is entirely grounded in their forms, and does not depend on any fact about the world⁵.

The Formality Thesis goes with a view of logic that seems quite natural. On this view, the role of logic is to help us make explicit *everything* on which an inference depends. When we have teased out hidden assumptions to the point where we have a formally valid argument, then we know that the process of explicitation has come to an end; we have made *all* of the assumptions on which the inference

⁴For worries of this kind, see B. BOLZANO, *Wissenschaftslehre*, §186; J. A. COFFA, «Machian Logic», *Communication and Cognition*, 8 (1975), 103–129; J. BARWISE and S. FEFERMAN, eds., *Model-Theoretic Logics* (New York: Springer-Verlag, 1985), 6; J. ETCEHEMENDY, «The Doctrine of Logic as Form», *Linguistics and Philosophy*, 6 (1983), 319–334; S. READ, «Formal and Material Consequence», *Journal of Philosophical Logic*, 23 (1994), 247–265.

⁵As stated, the Formality Thesis is ambiguous between a metaphysical thesis (there are inferences that are valid solely in virtue of their forms, and not at all in virtue of any facts about the world) and an epistemological thesis (there are inferences that we can know to be valid solely on the basis of knowledge of their forms, and independently of any knowledge of facts about the world).

depends explicit. We could not think of logic this way if the Formality Thesis did not hold, since there would always be further facts on which the validity of an inference depends, which could in principle be made explicit as further premises.

As natural as it may seem, though, the Formality Thesis is a substantive and controversial thesis. It does not follow immediately from the fact that inference (2) holds «in all matter»—that is, on all uniform substitutions of nonlogical terms—that it is good solely in virtue of its form, independently of any facts about the world. Although the schema

(4) X is prime. $X > 2$. Therefore, X is odd.

yields a good inference for every substitution for X , the goodness of these inferences is grounded in a fact about primes—namely, that every prime greater than 2 is odd. So clearly the inference from «every instance of the schema is a good inference» to «the instances are good solely in virtue of being instances of the schema» is not a good one in general. Maybe it is cogent when we restrict ourselves to schemata of a certain kind—those in which the only fixed terms are «logical constants». But if so, that is something that requires showing.

Which brings me, at last, to Abelard. Abelard interests me because, unlike later medieval logicians, he offers an argument for the Formality Thesis—a thesis his predecessors seem to have rejected. This argument motivates his distinction between formal and material consequence—or, in his terminology, perfect and imperfect entailment (*inferentia*⁶). He does not slide from «good in all matter» to «good in virtue of form»; indeed, he recognizes inferences that are good in all matter but depend for their validity on facts about the world. He gives a much subtler argument—and it is a good argument, in its philosophical context. But it is not an argument that can give *us* a good reason to accept the Formality Thesis.

2 Perfect and imperfect inferentia

An entailment is perfect, Abelard says, when

...from the structure (*complexio*) of the antecedent itself, the truth of the consequent is manifest, and the construction (*constructio*) of the antecedent is

⁶In translating *inferentia* as «entailment», I follow C. MARTIN, «Logic», in *The Cambridge Companion to Abelard*, ed. J. E. BROWER and K. GUILFOY (Cambridge: Cambridge University Press, 2004), 158–199.

so disposed that it contains also the construction of the consequent in itself, just as in syllogisms or in conditionals which have the form of syllogisms. (253.31-254.1)⁷

For example,

- (5) If every man is an animal and every animal is alive, every man is alive. (254.35)⁸

An entailment is imperfect, by contrast, when the connection between antecedent and consequent takes its necessity «from the nature of things» (*ex rerum natura*, 255.7-8), not from the construction of the antecedent and consequent, as in

- (6) If every man is an animal, every man is alive. (255.3)

Both perfect and imperfect entailments require a necessary connection between antecedent and consequent—indeed, the sense of the consequent must be *contained in* the sense of the antecedent (283.37-284.8). The difference is not in the strength of the modal connection (255.12-13), but in its ground.

Given these definitions, the thesis that some entailments are perfect amounts to the Formality Thesis⁹.

3 The dialectical background

Abelard's discussion of perfect and imperfect entailments takes place in the section of the *Dialectica* entitled «Of Topics». That may seem surprising, since in Aristotle

⁷All parenthetical references are to P. ABELARD, *Dialectica*, ed. L. M. DE RIJK (Assen: Van Gorcum, 1956). Unless noted, translations from Abelard are my own.

⁸Although Abelard is aware of the difference between arguments and conditionals, he applies the concepts *inferentia* and *consequentia*, as well as the perfect/imperfect distinction, to both (giving examples in both forms).

⁹Probably in its epistemological variant. Abelard holds that the truth of «if it is man, it is animal» does not depend on the existence of either man or animal: like all true consequences, it is an eternal truth (279.18). But if man and animal did not exist, then (as will be explained later in this essay) there would be no *locus differentia* and hence no Topical grounding. So the fact that man is species of animal cannot be the cause of the entailment (*consecutio*) but only its proof (*probatio*) (265.10-12). This suggests that what distinguishes perfect inferences from imperfect ones is a special epistemic character: their validity can be known independently of all knowledge about the world.

the topics are means for the discovery of valid syllogisms, rather than grounds for their validity¹⁰. However, by late antiquity it had become standard to conceive of the Topical maxims as axioms on which the validity of arguments might rest. In *De Topicis Differentiis* Boethius defines a maximal proposition as a «maximal, universal, principal, indemonstrable, and known per se proposition, which in argumentation gives force to arguments and to propositions»¹¹. For example, the maxim

(7) Whatever is present to the genus is present to the species

«supplies force to» the argument below and «makes [it] complete from without» (1188B-C):

(8) Every virtue is advantageous. Justice is a virtue. Therefore, justice is advantageous.

Commentators have wondered how the maxim here can play the role Boethius assigns it, of supplying force to the argument and completing it from without, when (8) has the form of a valid categorical syllogism¹². But the problem only arises if we assume that valid categorical syllogisms are distinguished from other forms of arguments by the fact that they require no external validation. Boethius shows no sign of accepting this view.

¹⁰For this view of Aristotle's *Topics*, see J. ALLEN, «The Development of Aristotle's Logic: Part of an Account in Outline», ed. J. J. CLEARY and W. C. WIANS, *Proceedings of the Boston Area Colloquium in Ancient Philosophy*, 11 (1995), 189; E. STUMP, *Boethius's De Topicis Differentiis* (Ithaca: Cornell University Press, 1978), 168–177; N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*. (Munich and Vienna: Philosophia Verlag, 1984), 23. A contrasting view understands Aristotelian topical maxims as quasi-logical laws that ground the validity of the inferences to which they are applied: J. BRUNSCHWIG, *Aristote: Topiques*, vol. I (Paris: Société d'Édition «Les Belles Lettres», 1967), pp. xl-xli; W. A. DE PATER, «La fonction du lieu et de l'instrument dans les Topiques», in *Aristotle on Dialectic: The Topics (Proceedings of the Third Symposium Aristotelicum)* (Oxford: Oxford University Press, 1968), 166, 174. But Aristotle does not seem to think that the acceptability of the particular syllogisms that fall under a topic depends on the acceptability of the general rules he gives; if he did, it would be difficult to make sense of the fact that he often acknowledges counterexamples to the maxims (e.g. at *Top.* 115b14, 117a18, 117b14, 121b30, 123b17, 124b19, 128b6).

¹¹E. STUMP, *Boethius's De Topicis Differentiis*, 1185B. This approach to the Topics goes back at least to Themistius S. EBBESEN, *Commentators and Commentaries on Aristotle's Sophistici Elenchi*, vol. 1 (Leiden: E. J. Brill, 1981), 118.

¹²N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*, 68–9; cf. E. STUMP, *Boethius's De Topicis Differentiis*, 183–4; P. ABELARD, *Dialectica*, 257.34-258.9

Sten Ebbesen claims that for Boethius as for Galen, «every inference owes its cogency to an axiom»:

The implication of the Boethian theory would seem to be that all proof proceeds, implicitly or explicitly, by instantiation and detachment and, as some medievals saw, that a categorical syllogism is not anything *sui generis*, as it depends on a law of inference of the same type as the ones that licence inferences involving other relations than plain predication.¹³

It is not clear that Boethius himself accepts all these implications. But eleventh- and twelfth-century logicians, whose main sources for syllogistic theory were the works of Boethius, embrace them explicitly. In his *Dialectica*, Garlandus Compotista says that the theory of Topics is prior to the theory of categorical and hypothetical syllogistic, and all syllogisms are ratified by topical maxims (*per maximam propositionem sillogismus approbatur*).¹⁴ For instance, the syllogism

(9) Every animal is a substance. Every man is an animal. Therefore, every man is a substance.

depends on the maximal proposition

(10) that which is universally attributed to the whole is [also universally attributed] to the part (*quod universaliter attribuitur toti, et parti*).¹⁵

More generally, «categorical syllogisms are aided by the Topics from the whole and from the part and from an equal» (114.18)¹⁶.

¹³S. EBBESEN, «Ancient Scholastic Logic as the Source of Medieval Scholastic Logic», in *The Cambridge History of Later Medieval Philosophy*, ed. N. KRETZMANN, A. KENNY, and J. PINBORG (Cambridge: Cambridge University Press, 1982), 112.

¹⁴G. COMPOTISTA, *Dialectica*, ed. L. M. D. RIJK (Assen: Van Gorcum, 1959), 86.13.

¹⁵Ibid., 92.29.

¹⁶Garlandus was anticipated in this view by Abbo of Fleury (945-1004) and other early commentators on the Boethian Topics (144, 152) N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*. Green-Pedersen summarizes the pre-1100 works by saying that they take the Topics to be an «... 'underlying logic' which shows or explains why the arguments are valid». (160). See also E. STUMP, *Dialectic and Its Place in the Development of Medieval Logic* (Ithaca: Cornell University Press, 1989), 87; E. STUMP, «Topics: Their Development and Absorption into Consequences», in *The Cambridge History of Later Medieval Philosophy*, ed. N. KRETZMANN, A. KENNY, and J. PINBORG (Cambridge: Cambridge University Press, 1982), 277.

The early twelfth-century works on the Topics collected in *Logica Modernorum*¹⁷ seem to follow Garlandus in taking all syllogisms to be validated by topical maxims. The *Introductiones dialectice Berolinenses*, for instance, takes all syllogisms in the mood Barbara to be licensed by the following topical maxim:

- (11) If something is predicated universally of something, then if something else is predicated universally of the predicate, that same thing is predicated universally of the subject.¹⁸

Abelard's picture of the Topics is largely taken over from this Boethian tradition. The function of a Topic, according to Abelard, is to confer inferential force on an entailment by grounding it in a real relation among the things to which its terms refer (256.35-257.1). For example, the conditional «if it is a man, it is an animal» is justified by the Topic from species, since man is a species of animal, and we know that genus necessarily applies to species (257.4-5). Following Boethius, Abelard takes a Topic to have two components: a *locus differentia* and a *maxima propositio*. The *locus differentia* (henceforth Differentia) is «that thing in the relation of which to something else the soundness of the entailment consists» (*ea res in cuius habitudine ad aliam firmitas consecutionis consistit*, 263.7-8). Although the Differentiae are things, not relations, they count as Topical Differentiae only insofar as they stand in relations to other things¹⁹ In the example, the Differentia is man, which stands in the species relation to animal. The *maxima propositio* (henceforth maxim) is a general proposition justifying an inference from an antecedent proposition containing a term for the Differentia to a consequent proposition containing a term for the thing to which it is related. In the example, the maxim is «of whatever the species is predicated, so is the genus» (*de quocumque praedicatur species, et genus*, 263.18).

Abelard's strikingly original move is to insist that some entailments do not stand in need of topical grounding at all (. . . *quia ita in se perfectae sunt huiusmodi inferentiae ut nulla habitudinis natura indigeant, nullam ex loco firmitatem habent*, 256.34-5). Perfect entailments, he says, do not «take their truth. . . from the nature of things» (256.21-2). A sign of this independence from things, Abelard claims, is that perfect consequences remain true in «. . . whatever terms you substitute» (255.32-3), whereas an imperfect consequence «depends on the nature of things» and does not

¹⁷L. M. DE RIJK, ed., *Logica Modernorum* (Assen: Van Gorcum, 1962–7).

¹⁸E. STUMP, *Dialectic and Its Place in the Development of Medieval Logic*, 116.

¹⁹N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*, 167

«remain true in any terms whatsoever, but only in those which preserve the nature of the entailment» (356.8-10). For example, the entailment in «if it is man, it is animal» can be destroyed by replacing «man» or «animal» with «stone» (356.15-19).

Therefore those consequences are correctly said to be true from the nature of things of which the truth varies together with the nature of things. But those [consequences] of which the construction preserves its necessity equally in any things at all, no matter what relations they have, take their truth from the construction (*complexione*), not from the nature of things. . . (256.20-23)

This is all that later medieval writers typically say about the distinction between formal and material consequence: formal consequences hold «in all terms». But Abelard cannot stop here, for as we have seen, the dominant view at the time he is writing—and a view he explicitly attributes to Boethius and Porphyry (257.32-258.13)—is that categorical syllogisms and other perfect entailments are grounded in Topics. A proponent of such a view could grant that syllogisms preserve validity in all substitution instances, and maintain either that

1. for each instance, there is a Topic grounding the entailment in some specific relation that holds between the things it concerns, or that
2. there is a single Topic that grounds *all* of the instances in some very general relations that hold between things.

Abelard offers arguments against both approaches (258-262 in his treatment of inferences, 352-365 in his treatment of conditionals). It is a measure of the success of these arguments, I think, that they do not get repeated: it becomes customary in later medieval manuals to infer from an inference's being good «in all terms» to its being good «in virtue of its construction» and not in virtue of the nature of things. But as we have seen, Abelard cannot take this inference for granted. Indeed, he does not even think that it is unrestrictedly valid. He claims that the consequence

(12) If it is alive, it is alive,

which certainly holds in all substitution instances, is not perfect in its construction (*ad inferentis constructionem*): one would have to add the premise «. . . and everything

that is alive is alive» (255.19-27)²⁰. Evidently, then, there is more to perfection than mere preservation of validity «in all terms».

Abelard makes this point explicitly in his discussion of the hypothetical syllogisms. Boethius had taken certain instances of what we now call «affirming the consequent» to be valid by virtue of «the nature of the things, in which alone these propositions can be asserted»²¹. For example, in the inference

(13) If it is not *a*, it is *b*; but it is *a*; thus it is not *b*,

Boethius claims, the major premise can only be true when the terms *a* and *b* are *contraries*, like «day» and «night». But when *a* and *b* are contraries, and it is *a*, then it follows that it is not *b*. Hence the inference is valid for all substitution instances in which the premises are true. In fact, Abelard thinks Boethius' claim that «if it is not *a*, it is *b*» can only be true when *a* and *b* are contraries is simply *wrong*: «if it is not *a*, it is *b*» can be true, he notes, when *a* is «animal» and *b* is «non-man» (499). But he goes on to say that, *even if Boethius were right* that *a* and *b* had to be contraries, and thus that no formal counterexample to the inference could be given, this fact would not show that such inferences are syllogisms (and hence perfect entailments):

Even if it were possible, whenever the consequent were affirmed, necessarily to affirm the antecedent *from any property whatever*—nevertheless there would be no form of syllogism in which, the consequent having been affirmed in this way, one could affirm the antecedent, or the antecedent having been denied, one could deny the consequent, since the entailment of a syllogism is supposed to be so perfect that no relation of things pertains to it. (502.19-25).

To say that an entailment is «perfect» is to say that our knowledge of its validity is completely independent of our knowledge of «the nature of things». Even if Boethius were right that (13) held in all terms for which the premise could be true, that would not be something we could know without knowing something about «the nature of things»—the relations of contrariety between *a* and *b*. An entailment that holds in all terms, then, need not be good in virtue of its construction.

²⁰This means that the one-premise conversion inferences necessary for the reduction of second- and third-figure syllogisms to the first figure cannot count as *perfect*. It seems odd that the validation of second- and third-figure syllogisms, which are perfect in Abelard's sense, should require the use of an imperfect inference. Does Abelard ever discuss this issue?

²¹L. OBERTELLO, *De hypotheticis syllogismus di A.M. Severino Boezio: Testo, traduzione, Logicalia; testi classici di logica* (Paideia, 1969), II.ii.4-5

4 Abelard's arguments that syllogisms are perfect entailments

Let us now consider Abelard's arguments for the claim that syllogisms do not have Topical grounding. Recall that there are two ways in which one might oppose Abelard's claim. First, one might argue that the validity of each individual syllogism is grounded in a particular relation between things (the local strategy). Second, one might argue that there is a single, very general relation between things that grounds the validity of *all* syllogisms in a particular mood (the global strategy). Abelard shows that neither approach will work. In my discussion, I will consider only categorical syllogisms, though Abelard brings similar considerations to bear on hypothetical ones.

4.1 The local strategy

Given a particular categorical syllogism, the obvious place to look for a Topical Differentia is in the middle term. For example, in the syllogism

(14) All animals are alive. All men are animals. Therefore, all men are alive.

one might naturally take «animal» to be the Differentia and apply the Topic «from the genus», with the maxim «whatever is predicated of the genus is also predicated of the species». But as Abelard points out, this Topic would only explain the entailment from the second premise to the conclusion, not the entailment from both premises together (258.14-17; cf. 356.4-11).

Even this kind of Topical grounding will be impossible when syllogisms have *false* or *accidentally true* premises, for example:

(15) Every body is colored. But everything sitting is a body. Therefore, everything sitting is colored (260.18-27).

In such a syllogism, «none of the propositions by themselves necessarily imply the conclusion» (260.19-20). For there is no real relation in the nature of things that could license the transition from either of these premises by itself to the conclusion²². Body, for instance, is not the genus of sitting thing, nor is colored

²²Similar considerations lead Abelard to claim that «if man is a species of stone, then if [something] is a man, it is a stone» is good in virtue of its construction (312). It could not take its necessity from «the nature of things», because in the nature of things man is *not* a species of stone. (312-3).

thing the genus of body²³. The only relation between terms to which we might appeal here is the relation of predication: colored is universally predicated of body, and body of sitting thing (cf. 259.1-9). But «A is universally predicated of B» might taken to express either

- (a) that A is asserted of all B (*secundum vocum enuntiationem*), or
- (b) that in the order of things, A is true of all B (*secundum rerum cohaerentiam*) (353.10-12; cf. 329.19-35).

If it means merely (a) that A is asserted of all B, then it clearly cannot ground a necessary entailment from «every C is B» to «every C is A»:

For who would concede that if «stone» were asserted universally of «man» in some assertion, whether true or false, the consequence which follows [i.e., «if every stone is an ass, then every man is an ass», 353.5] would be true? This is why we can assert «stone» (or anything else we like) of «man», but our assertion, which is manifestly false, confers no truth on the consequence. (353.15-19)

If, on the other hand, the relation «A is universally predicated of B» means that A is true of all B, then it is of no use in syllogisms with false premises, such as

- (16) All men are stones. All stones are asses. Therefore, all men are asses (353.5).

Nor is it of any use when it is merely accidental that A holds of all B, since entailment must be necessary (cf. 362.30-1). There are some categorical syllogisms, then, for which no local topical maxim can be found. And once we accept that one syllogism in Barbara holds in virtue of its construction, we might as well accept that all do (since all have the same construction).

4.2 The global strategy

If the validity of categorical syllogisms depends on a Topical maxim, then, it must be a maxim that captures the dependence of the conclusion on both premises. Syllogisms in Barbara, for instance, might be thought to depend on the rule:

²³Abelard says at 285.20-29 that «if it is body, it is colored» is only accidentally true.

(17) If B is predicated of A universally and C is predicated of B universally, then C is predicated also of A universally²⁴,

where «predicated of» is taken *secundum rerum cohaerentiam*. Might (17) be a Topical maxim that gives syllogisms in Barbara their inferential force?

Abelard's strategy here is to argue that (17), while perhaps a true *regula* (*regula*), is not a Topical *maxim*, because it lacks a corresponding Differentia (261.34-5, 265.25-266.2)²⁵. The argument that (17) lacks a Differentia is basically the same as the argument (rehearsed above) that particular syllogisms lack a Differentia. The Differentia would have to be some thing (*res*) that is predicated universally of some term in the conclusion. The only obvious candidate is the middle term (B). But the fact that B is predicated of all A could at best explain the entailment from one premise of the syllogism to the conclusion (from «every B is C» to «every A is C»), not the entailment from both premises to the conclusion. And it explains this only if B is predicated of all A truly and necessarily: that is, only if A and B stand in some beefier relation than mere predication—say, genus and species (362.26-31). This will not be the case for all syllogisms in Barbara.

Why should it matter whether or not (17) has a corresponding Differentia and is thus a genuine maxim? Here Abelard is not as explicit as he might have been, but I think we can reconstruct his reasoning. He is trying to show that syllogisms are grounded in their construction alone, not in «the nature of things». Apparently, he takes the fact that syllogisms do not depend on any genuine maxims to be sufficient grounds for this claim. Thus, although he does not deny that (17) is true if and only if the syllogism

(18) All A are B. All B are C. Therefore, all A are C.

is valid²⁶, he denies that this equivalence shows that our knowledge of the syllogism's validity depends on how things are in the world. In order to understand

²⁴I have used schematic letters to make the principle clearer. Abelard uses pronouns: «si aliquid praedicatur de alio universaliter et aliud praedicatur de praedicato universaliter, illud idem praedicatur et de subiecto universaliter» (261.14-16). There is a corresponding principle for hypothetical syllogisms: «si aliquid infert aliud et id quod inferat existat, id quoque quod infertur necesse est existere» (261.25-6).

²⁵E. STUMP, *Dialectic and Its Place in the Development of Medieval Logic*, 96; N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*, 197

²⁶In this respect, (17) fares better than an alternative *regula*, (CS*): «If B is predicated of A universally, then if C is predicated of B universally, then C is predicated also of A universally» (*si aliquid praedicatur de aliquo universaliter, tunc si aliud praedicatur de praedicato universaliter, et de subiecto,*

Abelard's reasoning here, we need to understand why he thinks that only a genuine Topical maxim—one with a Differentia—can ground the entailment in «the nature of things».

I propose that Abelard is thinking along the following lines. A Topical maxim gives a rule for inference that is based on its Differentia: that is, on some thing (*res*) in the world²⁷. The inferential force (*vim inferentiae*) which a maxim brings to an imperfect inference comes from the relation in which the Differentia stands to a term in the conclusion of the inference (*ex habitudine quam habet ad terminum illatum*, 256.36-7). For example, in the valid consequence «if it is man, it is animal», the inferential force comes from the relation (species) in which the Differentia (man) stands to animal. The Differentia, then, is the thing (*res*) in the nature of which the validity of imperfect inferences is grounded²⁸. A *regula* without a Differentia, then, although it might still be thought to ground the validity of inferences, could not ground it in «the nature of things», as a maxim does.

To modern eyes, this reasoning appears to make an unwarranted assumption: that the totality of facts about «the nature of things» is exhausted by facts of the form

- (19) A is F, or
- (20) A stands in the relation R to B.

Given this assumption, it follows from (17)'s lack of a Differentia that (17) is not a fact about «the nature of things» and must therefore depend for its truth on something else: the construction or form of the syllogism, the way it is put together in thought and language. But if we relax the assumption and count as facts about «the nature of things» facts with more logical complexity, such as

352.31-3). (17) and (CS*) are not equivalent, because the law of exportation fails in Abelard's logic. In fact, Abelard argues, (CS*) and the corresponding *regulae* for other syllogistic moods have many false instances (358.34-362.17).

²⁷N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*, 167. In the consequence «if it is man, it is animal», the *locus differentia* is *man*; when Abelard calls the Topic «from species», giving the *relation* in which the Differentia stands to something else, he is saying «from where the locus comes» (*unde sit locus*, 264.5-34). Green-Pedersen conjectures, plausibly, that Abelard insists that the Differentia be a *thing* and not the relation itself because the latter approach would make the relations (e.g., genus, species) into «independent realit[ies]» and contradict his nominalism (168).

²⁸Cf. 255.7-9, on the consequence «if every man is animal, every man is alive»: «These inferences, although they are imperfect in the construction of the antecedent, nonetheless most often take their necessity *from the nature of things*, just as with [the consequence] which we put down earlier from *animal to alive*, since *the nature of animal*, in which as a substantial form *alive* inheres, never allows animal itself to exist without life».

- (21) A, B, and C stand in the relation Q, or
 (22) not both: { (all A are B and all B are C) and not (all A are C) }, or even
 (23) for all A, B, and C: A, B, and C stand in the relation Q,

then there is no longer any reason to think that (17) is not a fact about «the nature of things», and consequently no reason to think that syllogisms in Barbara do not depend on facts about the world: more general facts, to be sure, than most Topically grounded inferences, but no less facts about «the nature of things». Granted, the entailment in a categorical syllogism cannot depend on the real relation of one thing to another; but might it not depend on some more complex feature of the world?

This question would become acute for Kant—for whom «the nature of things» consists of just the kind of complex, generalized relational facts Abelard does not consider (e.g., the laws of Newtonian science)—and even more pressing for Frege, Russell, and Wittgenstein, whose new logical notation allowed the question to be raised in a more explicit way. But Abelard doesn't answer it. He is not even in a position to ask it. In order to do so, he would have to reject the broadly Aristotelian assumption he inherits from his sources and shares with all of his contemporaries, that all facts about the world can be described by predicating «something of something» (*ti kata tinou*)²⁹. Given that assumption, Abelard is right to deny that syllogisms depend for their validity on facts about the world.

Indeed, the same reasoning that leads Abelard to this conclusion should lead him to accept the inference

- (24) A is east of B. B is east of C. Therefore A is east of C.

as valid in virtue of its construction³⁰. For suppose the premises were false. What would be the Differentia? Since the inference is not valid in virtue of B's relation to something else, Abelard would reason, it must not be valid in virtue of «the nature of things».

This point reveals the extent to which Abelard's arguments for the formality of syllogisms are unavailable to us today. Abelard would have to concede that (24) is valid in virtue of its construction, while

²⁹«According to Abelard, if a statement of the form xRy is true, then what makes it true is nothing but individual subjects and their monadic properties» J. E. BROWER, «Abelard's Theory of Relations: Reductionism and the Aristotelian Tradition», *Review of Metaphysics*, 51 (1998), 623.

³⁰I am not aware of any passages in which Abelard discusses such inferences.

(25) A is a donkey. Therefore, A is an animal.

is valid in virtue of the nature of donkeys. No modern advocate of the Formality Thesis, I take it, would make a principled distinction between these two cases. Similarly, as we have seen, Abelard takes syllogisms in Barbara to be valid in virtue of their construction, while denying the same status to

(26) If A then B. If not B, then not A.

or

(27) A is alive. A is alive.

Again, his views about the basis for the Formality Thesis—views we do not share—would make a distinction of principle where we see none.

5 Conclusion

Unlike later medieval logicians who make a distinction between formal and material consequence, Abelard explains why it is important to distinguish between perfect and imperfect entailments. He argues that the dominant view, on which all inferences are grounded in topical maxims, cannot be sustained, and that we must recognize some inferences as not needing external grounding. This is the first argument for the Formality Thesis of which I am aware.

Abelard's arguments seem to have been persuasive: the majority of Abelard's twelfth-century successors distinguish between «arguments which rest upon loci [Topics] (*locales*) and those that are valid by their form (*complexionales*)»³¹ The distinction persists in the thirteenth century and is a likely ancestor of the fourteenth-century (continental) distinction between formal and material consequence³². But we no longer find arguments for the Formality Thesis that would support the distinction. The reason, perhaps, is that there is no longer a concerted opposition.

³¹N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*, 200

³²W. KNEALE and M. KNEALE, *The Development of Logic* (Oxford: Oxford University Press, 1962), 274–5, 279; E. STUMP, *Dialectic and Its Place in the Development of Medieval Logic*, 127; N. J. GREEN-PEDERSEN, *The Tradition of the Topics in the Middle Ages: the Commentaries on Aristotle's and Boethius' «Topics»*, 198

After Abelard, it is taken for granted that valid inferences divide into those whose validity can be attributed to their structure and those whose validity depends on their terms and the nature of the things to which they refer.

It is tempting for contemporary advocates of the Formality Thesis to point to fourteenth-century logicians as predecessors. But if I am right about the philosophical basis of the medieval distinction, they should not do so. We cannot accept the premises of Abelard's argument for the Formality Thesis, so if we are going to accept some version of the Formality Thesis ourselves, it will have to be on other grounds³³.

³³This essay is partially derived from the appendix to my dissertation: J. MACFARLANE, «What Does It Mean to Say that Logic Is Formal?», 2000. It was delivered at ESMLS XIX in Geneva in June 2012.