1 Quine’s “three grades of modal involvement”

1. **predicate**: ‘is necessary’
   
   ‘10 > 9’ is necessary

2. **statement operator**: ‘it is necessary that’ + closed formula
   
   \( \square 10 > 9 \)

3. **sentence operator**: ‘it is necessary that’ + open formula
   
   \( \square x > 9 \)

Quine argues that the first grade is unproblematic, if logical necessity is meant (we have a clear understanding of that). If analytic necessity is meant—that is, truth in virtue of meaning—Quine has some worries about that \[8, 11\]. But the problems here aren’t really logical problems. In \[10, 9\] Quine assumes, for the sake of argument, that analyticity is unproblematic. His aim is to show that, even if he spots us analyticity, there are problems for modal logic.

The second grade is also unproblematic if it is understood in terms of the first grade: \( \square S \) is true iff \( S \) is necessary. However, it is misleading. If we are really talking about sentences (mentioning them), then we should use a predicate. (Quine argues that modal logic was “conceived in sin” \[10\] pp. 165–6]: the sin of confusing use and mention, and reading ‘\( \supset \)’—a sentential connective—as ‘implies’—a relation.) The second grade is dangerous, because it makes us think we understand things that can’t be explained in terms of the first grade: iterated modal operators (as in ‘\( \square \Diamond \square S \)’ \[10\] pp. 168–170] and quantifying in to modal contexts (as in ‘\( \forall x \square F x \)’).

The third grade is where things get really problematic. The only way to have it is to embrace what Quine calls “Aristotelian essentialism,” the view that necessity is not linguistic in origin, and that things have certain of their properties necessarily, others only contingently, independently of how they are described. This, Quine thinks, is hopeless; so, quantified modal logic must go, and since we don’t get anything from the second grade that we didn’t have in the first, modal logic might as well go.

2 Opaque contexts

An occurrence of a singular term in a statement is **purely referential** if “the term serves in
that particular context simply to refer to its object” [10, p. 160]. Frege’s criterion for referential occurrence is substitutivity of identicals. If you can’t always replace an occurrence of a term with another term that denotes the same object without changing the truth value of the whole, then that occurrence of the term is not purely referential.

A context is opaque if “it can render a referential occurrence [of a singular term] non-referential” [10, p. 161]. Examples of opaque contexts:

(1) “___” has six letters.
(2) ___is so-called because of his size.
(3) Lois believes ___is boring.
(4) Necessarily ___> 5.

3 Opaque contexts and quantifying in

At the core of Quine’s objection to quantified modal logic is the observation that, unlike terms like ‘Giorgione’, variables of quantification can only have purely referential occurrences. Consider

(5) \(x\) is so-called because of his size

The variable ‘\(x\)’ here ranges over objects (it’s not a substitutional quantifier). So we need to be able to say whether this open sentence is satisfied by various assignments of objects to ‘\(x\)’.

Okay, suppose we assign Giorgione Barbarelli da Castelfranco, the Italian painter, to ‘\(x\)’. Is the open formula true on that assignment?

You might say yes, because

(6) Giorgione was so-called because of his size.

(In Italian the suffix -one/-ona denotes largeness.)

But you might equally say no, because

(7) Barbarelli was not so-called because of his size.

So there is no good way to answer the question whether the open sentence (5) is true on an assignment that assigns Giorgione (= Barbarelli) to \(x\).

This shows, Quine thinks, that it doesn’t make sense to quantify into “so-called” contexts. Names (like ‘Giorgione’) can have occurrences that aren’t purely referential, but variables can only be purely referential, and can’t occur in opaque contexts.
Extra credit: Could we evade the difficulty by thinking of our domain as consisting not of regular objects, but of pairs of objects and names? Then we could take (5) to be true of the pair (Giorgione, ‘Giorgione’) and false of the pair (Giorgione, ‘Barbarelli’).

If we did this, how would you state the conditions for ‘x kicked y’ and ‘x is so-called’ to be satisfied by an assignment of values to the variables?

What should we do with ‘There are at least two people who are so-called because of their size?’

4 Quine and Smullyan on the number of planets

Quine uses this argument to convince us that (8) is an opaque context:

1. Necessarily 9 > 5.
2. 9 = the number of planets.
3. So, necessarily the number of planets > 5.

Quine thinks that the conclusion is clearly false, and both premises clearly true.¹ So the argument is invalid. By Frege’s criterion, this shows that (4) is an opaque context. Since it doesn’t make sense to have an (objectual) variable in an opaque context, we can’t make sense of

(8) □x > 5

and so we can’t make sense of

(9) ∃x□x > 5

... necessity does not properly apply to the fulfillment of conditions by objects
(such as the ball of rock which is Venus, or the number which numbers the planets), apart from special ways of specifying them. [9] p. 151

Smullyan [13] thinks that Quine’s argument can be met.² Smullyan assumes, with Russell, that definite descriptions can be understood as quantifiers. But that means they have scopes. He thinks that Quine’s argument ignores this, and trades on a scope ambiguity (see [13] p. 33] for the charge).

Using modern generalized quantifier notation, we can put Smullyan’s point like this. The following argument is valid:

¹At the time Quine wrote, Pluto was considered a planet.
²In reading Smullyan’s article, you’ll see that he uses an old-fashioned notation, the notation of Russell and Whitehead’s Principia Mathematica. In this notation, dots sometimes mean conjunction and are sometimes used instead of parentheses to indicate grouping. Subscripted variables are sometimes used for quantification. For an explanation of how to read this notation, see http://plato.stanford.edu/entries/pm-notation/
1. $\Box 9 > 5$
2. $\text{the}_x (x \text{ numbers the planets}, x = 9)$
3w. $\text{the}_x (x \text{ numbers the planets}, \Box x > 5)$

But this one is invalid:

1. $\Box 9 > 5$
2. $\text{the}_x (x \text{ numbers the planets}, x = 9)$
3n. $\Box \text{the}_x (x \text{ numbers the planets}, x > 5)$

The difference between them is the relative scope of the quantifier and the modal operator in the conclusion. When the quantifier takes wide scope (3w), the argument is valid and the conclusion is true. When it takes narrow scope (3n), the argument is invalid and the conclusion false.

**Exercises:**

4.1 Give a natural deduction of the argument from (1) and (2) to (3w). Try to see why you can’t get (3n) from these premises.

Smullyan takes these considerations to show that, in applying Frege’s “substitutivity” criterion for referential occurrences of terms, we should restrict ourselves to names. If you treat definite descriptions as names, ignoring their scopes, then you will run into paradoxes, but “the modal paradoxes arise not out of any intrinsic absurdity in the use of modal operators but rather out of the assumption that descriptive phrases are names” [13, p. 34].

Quine acknowledges the point [10, p. 173]. And he doesn’t think it would help to use examples with proper names, because on Quine’s view names can be definitionally reduced to definite descriptions (Pegasus = the thing that Pegases). But he thinks we can see the problem in pure quantification theory, without names or descriptions. For it’s a theorem of first-order logic with identity that

\[(10) \forall x \forall y (x = y \supset \phi x \equiv \phi y)\]

for any open formula $\phi$. So, let $\phi_-$ be $\Box x = _-$. Then

\[(11) \forall x \forall y (x = y \supset \Box (x = x) \equiv \Box x = y)\]

But

\[(12) \Box x = x\]

is a theorem. From (12) and (11) we can derive

\[(13) \forall x \forall y (x = y \supset \Box x = y)\]
5. The slingshot argument

The upshot is that, if it makes sense to quantify inside modal contexts, we must accept that if \(x = y\), then \(\Box x = y\).

Quine thinks this is unacceptable, since there are clearly some true identities that are true only contingently. For example, Hesperus = Phosphorus. Quine is willing to say that Hesperus is necessarily Hesperus, but how could it necessarily be Phosphorus, when it took real empirical effort to discover this? He continues:

There is yet a further consequence, and a particularly striking one: Aristotelian essentialism. This is the doctrine that some of the attributes of a thing (quite independently of the language in which the thing is referred to, if at all) may be essential to the thing, and others accidental. \[10\] pp. 175–6

Formally, for some \(F, G\):

\[
\exists x (Fx \land Gx \land \Box Fx \land \neg \Box Gx)
\]

Quine calls this the “metaphysical jungle of Aristotelian essentialism.” He’s expressing here his view that necessity, if it makes sense at all, is rooted in language. When we get to Kripke (next time), we’ll see him embracing both the necessity of identity and Aristotelian essentialism.

5 The slingshot argument

Quine uses another technical argument in \[10\] p. 163 and \[9\] p. 159. This argument has come to be called “the slingshot,” in recognition of its small size and giant-slaying potential.

The argument purports to show that any context that admits both (a) substitution of identicals and (b) substitution of logical equivalents is truth-functional. Quine takes this to be an impossibility proof for quantified modal logic: it shows that any sentential operator that allows substitution of logical equivalents must either be create an opaque context (in which case quantified modal logic is off the table) or be truth-functional (so that the modal logic is trivialized).

Here we give a form of slingshot that differs from the version in \[9\] in using definite descriptions rather than class abstracts. The basic idea of a slingshot argument was first proposed by Alonzo Church \[2\]. (The presentation here is indebted to \[7\].)

We assume that the following rules are valid for the \(\Box\) operator:

**Substitution of logical equivalents (Equiv)** If \(\phi\) and \(\psi\) are logically equivalent, then \(\Box \psi\) may be inferred from \(\Box \phi\).

**Substitution of co-denoting definite descriptions (Coden)** From \(\Box t_1 = t_2\) and \(\phi\), where \(t_1\) and \(t_2\) are definite descriptions, \(\phi^{t_2/t_1}\) may be inferred, where \(\phi^{t_2/t_1}\) is the result of substituting \(t_2\) for some or all of the occurrences of \(t_1\) in \(\phi\).
Then we argue as follows for any sentences $\phi$ and $\psi$:

1. $\phi \land \phi$
2. $\Box \phi$
3. $\Box (a = \forall x (x = a \land \phi))$  \hspace{1cm} \text{Equiv, 2}
4. $\forall x (x = a \land \phi) = \forall x (x = a \land \psi)$  \hspace{1cm} \text{(provable from 1)}
5. $\Box (a = \forall x (x = a \land \psi))$  \hspace{1cm} \text{Coden, 3, 4}
6. $\Box \psi$  \hspace{1cm} \text{Equiv, 5}
7. $\Box \psi$
8. $\vdash$  \hspace{1cm} \text{(as above)}
9. $\Box \phi$
10. $\Box \phi \equiv \Box \psi$  \hspace{1cm} \equiv \text{Intro, 2–9}

This argument establishes the conditional

$$(\phi \land \psi) \supset (\Box \phi \equiv \Box \psi).$$

We have shown that if $\phi$ and $\psi$ are both true, then $\Box \phi$ and $\Box \psi$ have the same truth value. This goes only part way towards establishing that ‘$\Box$’ is truth-functional:

$$(\phi \equiv \psi) \supset (\Box \phi \equiv \Box \psi).$$

To establish truth-functionality, we also need to show that if $\phi$ and $\psi$ are both false, $\Box \phi$ and $\Box \psi$ have the same truth value. We could do that by means of the following argument [3 p. 441]:

...
6. The Gödel slingshot

Gödel's version of the argument, given in [5], does not use Equiv. Instead, it uses a weaker principle,

**Gödel equivalence (Gödel)** From \(\Box \mathbf{\phi} \wedge \Box \mathbf{\psi}\) and \(\Box (\mathbf{\phi} \equiv \Box \mathbf{\psi})\), Gödel equivalence (Gödel) From \(\Box \mathbf{\phi} \wedge \Box \mathbf{\psi}\) and \(\Box (\mathbf{\phi} \equiv \Box \mathbf{\psi})\), may be inferred, and vice versa (where \(\Phi\) is a predicate and \(\alpha\) a term).

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\[
\begin{array}{c|l}
1 & \neg \phi \wedge \neg \psi \\
2 & \Box \phi \\
3 & \Box \neg (a = \iota x (x = a \wedge \neg \phi)) \quad \text{Equiv, 2} \\
4 & \iota x (x = a \wedge \neg \phi) = \iota x (x = a \wedge \neg \psi) \quad \text{(provable from 1)} \\
5 & \Box \neg (a = \iota x (x = a \wedge \neg \phi)) \quad \text{Coden, 3, 4} \\
6 & \Box \psi \quad \text{Equiv, 5} \\
7 & \Box \psi \\
8 & \vdash \quad \text{(as above)} \\
9 & \Box \phi \\
10 & \Box \phi \equiv \Box \psi \quad \equiv \text{Intro, 2–9}
\end{array}
\]

**Exercise:**

5.1 Prove that ‘\(P\)’ is logically equivalent to ‘\(a = \iota x (x = a \wedge P)\)’. Use the Russellian equivalences from the handout on Generalized Quantifiers. (Remember: to prove logical equivalence, you need to prove a biconditional using no premises.)

5.2 Show that (4) in the proof of the slingshot can be derived from (1) using standard logical rules and Russell’s definition. (Note that the definite descriptions take narrow scope.)

5.3 Suppose we wanted to say (against Russell) that when there is not a unique \(x\) such that \(Fx\), ‘\(G(\exists x Fx)\)’ is neither true nor false. Would ‘\(P\)’ still be logically equivalent to ‘\(a = \iota x (x = a \wedge P)\)’? Why or why not? (Careful: In this case the Russellian equivalence could not be used. You might try thinking of rules for \(\iota\) that would still be valid. What would validity mean in such a system?)
7 Applications of slingshot arguments

I've presented the slingshot arguments with the modal box ‘□’, and thus as arguments against quantified modal logic. These arguments show that any modal box that obeys the principles Coden and Equiv, or Coden and Gödel, gets trivialized.

There are many other applications of this argument, which rely on different readings of ‘□’:

• If we read \( \square \phi \) as ‘the fact that \( \phi = \text{the fact that } 2+2=4 \)', then we can get an argument that every fact is identical to the fact that \( 2+2=4 \)—that is, there is only one fact.

• If we read \( \square \phi \) as ‘Albert believes that \( \phi \)', then we can get an argument that if Albert believes one proposition, he believes every true proposition (see [12, pp. 148–9]).

In each case, the slingshot argument forces us to give up either substitution of logical equivalents or substitution of codenoting terms in the contexts in question.

Donald Davidson [4, pp. 41–2] reads \( \square \phi \) as ‘the statement that \( p \) corresponds to the fact that \( \phi \)', and uses the slingshot to argue that there is just one fact to which all true
7. Applications of slingshot arguments

statements correspond: “the Great Fact,” as he puts it. (Of course, he takes this to be a reductio of approaches to semantics that talk of correspondence to facts.)

When does (6) [“The statement that \( p \) corresponds to the fact that \( q \)’] hold? Certainly when \( p \) and \( q \) are replaced by the same sentence; after that the difficulties set in. The statement that Naples is farther north than Red Bluff corresponds to the fact that Naples is farther north than Red Bluff, but also, it would seem, to the fact that Red Bluff is farther south than Naples (perhaps these are the same fact). Also to the fact that Red Bluff is farther south than the largest Italian city within thirty miles of Ischia. When we reflect that Naples is the city that satisfies the following description: it is the largest city within thirty miles of Ischia, and such that London is in England, then we begin to suspect that if a statement corresponds to one fact, it corresponds to all. (‘Corresponds to the facts’ may be right in the end). Indeed, employing principles implicit in our examples, it is easy to confirm the suspicion. The principles are these: if a statement corresponds to the fact described by an expression of the form ‘the fact that \( p \)’, then it corresponds to the fact described by ‘the fact that \( q \)’ provided either (1) the sentences that replace \( p \) and \( q \) are logically equivalent, or (2) \( p \) differs from \( q \) only in that a singular term has been replaced by a coextensive singular term. The confirming argument is this. Let ‘s’ abbreviate some true sentence. Then surely the statement that \( s \) corresponds to the fact that \( s \). But we may substitute for the second ‘s’ the logically equivalent (‘the \( x \) such that \( x \) is identical with Diogenes and \( s \)’ is identical with (the \( x \) such that \( x \) is identical with Diogenes)’). Applying the principle that we may substitute coextensive singular terms, we can substitute ‘\( t \)’ for ‘\( s \)’ in the last quoted sentence, provided ‘\( t \)’ is true. Finally, reversing the first step we conclude that the statement that \( s \) corresponds to the fact that \( t \), where ‘\( s \)’ and ‘\( t \)’ are any true sentences.

Since aside from matters of correspondence no way of distinguishing facts has been proposed, and this test fails to uncover a single difference, we may read the result of our argument as showing that there is exactly one fact. Descriptions like ‘the fact that there are stupas in Nepal’, if they describe at all, describe the same thing: The Great Fact. No point remains in distinguishing among various names of The Great Fact when written after ‘corresponds to’: we may as well settle for the single phrase ‘corresponds to The Great Fact’. This unalterable predicate carries with it a redundant whiff on ontology, but beyond this there is apparently no telling it apart from ‘is true’.

The difference between the Quine and Gödel slingshots is mainly interesting in connection with theories of facts and propositions. As [1] pointed out, logical equivalents can bring in new material that you might think becomes “part of” the proposition. For example, “John is sleeping and either Bert is awake or Bert is not awake” seems to be about Bert in a way that “John is sleeping” is not. The Gödel slingshot might be preferable, then, in arguing against theories of facts and propositions.
8 Critique of the slingshot

Note the role of definite descriptions in both forms of the argument. Recall that there are two schools of thought about definite descriptions: (1) they are singular terms, like names; (2) they are quantifiers.

- If definite descriptions are quantifiers, as Russell held, then easy to see why φ is logically equivalent to \( \forall a = \exists x (x = a \land \phi) \) and \( \neg \phi \) to \( \neg \forall a = \exists x (x = a \land \neg \phi) \) (the first part is a homework problem). But then substitution of codenoting definite descriptions is fishy, as Smullyan reminded us, unless the definite description takes wide scope. (And here the modal operator needs to take wide scope.)

- If definite descriptions are singular terms, as Frege held, then perhaps substitution of co-denoting definite descriptions is on firmer ground. But it is harder to see why our statements should be logically equivalent. (Another homework problem deals with this issue.)

References
